Important Suggestions

Please note the following points:

- All the question numbers mentioned below are taken from the examples of **Understanding Mathematics by M.L. Aggarwal**, with topic numbers mentioned alongside.
  For example:
  5.13 – 1(a), 2(b), 3(a): indicates the question nos. 1(a), 2(b) and 3(a) of the illustrative examples following the topic **5.13.**
  Ex 5.15 – 5: indicates question no. 5 of exercise 5.15
- The questions given below are to give you an idea of the TYPE of questions that might be asked from a particular topic in the exam. The below list is only indicative and not exhaustive.
- Do NOT memorize the solution of a particular problem. Understand the type and the concept behind it.
- Do NOT ignore the past years questions for any chapter.

Section A

Relations

Example 19. *Let* **Z** *be the set of all integers and R be a relation on* **Z** *defined as* $R = \{(a, b) : a, b \in \mathbf{Z}$ *and* $(a - b)$ *is divisible by* $5\}$. *Prove that R is an equivalence relation. Find the set of all elements of* **Z** *related to 1.*

Example 14. *Determine whether the relation R defined on the set* **R** *of all real numbers as* $R = \{(a, b) : a, b \in \mathbf{R}$ *and* $a - b + \sqrt{3} \in S$ *where S is the set of all irrational numbers\}, is reflexive, symmetric and transitive.*

Example 5. *Show that the relation R on the set* **R** *of real numbers, defined as* $R = \{(a, b) : a \leq b^2\}$, *is neither reflexive nor symmetric nor transitive.* (NCERT)

Other important examples: 7, 10, 12, 22, 25

Functions

If the function $f(x) = \sqrt{2x - 3}$ is invertible then find its inverse. Hence prove that $(fof^{-1})(x) = x$.

Let R^+ be the set of all positive real numbers and f: $R^+ \longrightarrow [4, \infty)$: $f(x) = x^2 + 4$. Show that inverse of f exists and find f^{-1}.

■ **Domain and Range:** 5, 7, 8, 10(ii), 11, 14, 15, 16

■ **Types of functions:** 4, 5, 9, 10, 12, 14, 15, 19, 20

■ **Composition of functions:** 2, 5, 7, 8, 10, 16, 20

■ **Invertible functions:** 6, 9, 12, 15, 16

Binary Operations

The binary operation $*: R \times R \rightarrow R$ is defined as $a * b = 2a + b$.

Find $(2 * 3) * 4$.

A binary operation * defined on Q-{1} is given by a * b = a+b – ab. Find the identity element.

Other examples: 3, 9, 13, 15, 18, 19, 24, 26

Inverse Trigonometric Functions

■ **Solving/Evaluate type:**

Solve: $3\tan^{-1}x + \cot^{-1}x = \pi$

Solve: $\sin^{-1}\cos(\sin^{-1}x) = \frac{\pi}{3}$

Other examples: **2.1** : 4, 11(ii) **2.2**: 6, 8(iii), 10(ii), 15(ii), 24, 26(ii), 41(iv, vi), 43(ii, iii, v), 45

■ **Proving type:**

If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, prove that $a + b + c = abc$.

Prove that $tan^{-1}\frac{1}{2} = \frac{\pi}{4} - \frac{1}{2}cos^{-1}\left(\frac{4}{5}\right)$.

Other examples: **2.2:** 11(ii), 12, 14(iii), 16, 18(ii), 19, 20, 22, 23(iii), 25(iv), 26(ii), 29, 32, 34(iv), 35(ii), 36, 38(ii), 40(iii), 46, 48

Determinants

■ **Type 1:** "*Without expanding at any stage, find the value of/evaluate*"

Without expanding at any stage, find the value of:

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

Without expanding at any stage, find the value of the determinant:

$$\Delta = \begin{vmatrix} 2 & x & y+z \\ 2 & y & z+x \\ 2 & z & x+y \end{vmatrix}$$

Other examples: **4.2:** 4, 5(ii), 6(i)

■ **Type 2:** "*Using properties of determinants, prove that/solve for x*"

Use properties of determinants to solve for x:

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0 \text{ and } x \neq 0.$$

Using properties of determinants, prove:

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x),$$ where p is any scalar.

Other examples: **4.2:** 8(i), 13(ii), 14(i), 15, 18(ii), 19, 20(ii), 23(iii), 24, 25(i), 29, 31(i), 32(ii), 33, 34, 36, 41, 43, 45

Matrices:

■ **2 marks type:**

If $A = \begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix}$ and A is symmetric matrix, show that $a = b$

Find the value of k if $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - kM - I_2 = 0$.

3

Other examples: **3.3:** 6, 8, 9, 18, 28. **3.4:** 5, 8, 12. **4.4:** 1, 9, 12 21, 25,

■ **4 marks type:**

Using elementary transformation, find the inverse of the matrix:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Evaluate $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

Hence , Solve the system of equations,

$$3x - 2y + 3z = 8, 2x + y - z = 1, 4x - 3y + 2z = 4.$$

Elementary operations: **3.5:** 2, 3(ii), 4

Solution of Linear Equations using Matrices – Martin's Rule: **4.5:** 6, 7, 9, 10, 13

Continuity:

Find the value of constant 'k' so that the function f(x) defined as:

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at $x = -1$.

Examples: **5.1.1 –** 5, 6, 12, 13, 15, 19, 23, 26. **5.1.2 –** 13, 14

Differentiability:

Prove that the function $f(x) = |x - 1|, x \epsilon R$, is continuous at x = 1 but not differentiable.

Show that the function $f(x) = \begin{cases} x^2, x \leq 1 \\ \frac{1}{x}, x > 1 \end{cases}$ is continuous at $x = 1$ but not differentiable.

Examples: 5.2.1 – 7, 10, 13, 14, 16, 17

Differentiation:

If $x = tan \left(\frac{1}{a} log\, y\right)$, $prove\ that\ (1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

If $y = e^{a cos^{-1} x}$, where $-1 \leq x \leq 1$ then show that:

$$(1 - x^2)y_2 - xy_1 - a^2y = 0$$

Examples: 5.4 – 7, 9 **5.5.1** – 4, 6(ii), 7(v), 10(i) **5.6** – 7, 12, 17, **5.7.3** – 5(ii), 10(i), 17, 19
5.8 – 3(i), 11, 14, 16 **5.9** – 4, 8, 12, 18 **5.11** – 7, 8, 9, 10, 12, 16, 29, 30

Mean Value Theorem:

Verify Rolle's theorem for the following function: $f(x) = e^{-x} \sin x \quad on \quad [0, \pi]$

Examples: 5.12 – 4, 5(b), 6(b) **5.13** – 1(a), 2(b), 3(a) **Ex 5.15 –** 5

L'Hospital Theorem:

Example 2. *Evaluate the following limits :*

(i) $\underset{x \to 0}{Lt} \dfrac{\sin x - x + \frac{1}{6}x^3}{x^3}$

More examples: **6.1** – 3, 6(i), **Ex. 6.1** – 11(i), **6.2** – 3(iii, iv), 5(ii) **6.3** – 4, 6(ii)

Application of Derivatives:

Find the approximate change in the volume 'V' of a cube of side x metres caused by decreasing the side by 1%.

Find the points on the curve $y = 4x^3 - 3x + 5$ at which the equation of the tangent is parallel to the x-axis.

Water is dripping out from a conical funnel of semi-verticle angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/sec in the surface, through a tiny hole at the vertex of the bottom. When the slant height of the water level is 4 cm, find the rate of decrease of the slant height of the water.

A cone is inscribed in a sphere of radius 12 cm. If the volume of the cone is maximum, find its height.

More examples:

7.1 – 4, 6, 9, 14, 19, 21, 27

7.2 – 4, 7, 10, 13, 19, 26, 31, 36, 37

7.3 – 1(iv), 3, 6, 8, 10

7.4.2 - 4, 8, 10, 14, 17, 19(iii), 25, 29, 32

7.7 - 2, 6, 9, 11, 12, 16, 18, 20, 22, 23, 25, 27, 28, 30, 34, 41,48, 49, 52, 53

Indefinite Integrals:

Evaluate : $\int \frac{x^3+5x^2+4x+1}{x^2} dx.$

Evaluate: $\int tan^{-1} \sqrt{x}\, dx$

Evaluate : $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$

More Examples:

8.2.3 – 3(i), 4(iii), 7(i), 10(i, ii), 12(i), 20

8.3 - 1(iii), 5(ii)

8.4 – 1(ii), 4(iii), 5(ii), 7(ii), 9, 13(ii)

8.4.1 – 4(ii), 5(ii), 9(ii)

8.4.2 – 2(iii), 3, A special type – 2, 3

8.5 – 5(ii), 6, 8, 9(ii), 10(ii), 11(i), Two special types – 4(ii), 5(i), 6(ii), 7

8.6 – 11(i), 13, 16(ii), 20(ii)

8.6.1 – 4(i), 6(ii), 7(i), 8(i), 9(ii), 11(i), 12(i)

8.6.2 – 5(ii), 6(ii) **8.6.3 –** 4

Definite Integrals:

Evaluate: $\int_0^{\pi/2} \frac{cos^2 x}{1+sinx\, cosx} dx$

Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$

Evaluate: $\int_1^3 (x^2 + x) dx$, expressing as a limit of sum.

More Examples:

8.7 – 4, 7

8.9 – 1(i), 2, 5(i), 6(iii), 9(i), 10(i, ii), 14(ii), 15(ii, iv), 18, 19(ii), 20(i), 21(i), 24(ii), 26(i), 29(ii)

Differential Equations:

■ **Formation of Differential equations:**

Find the differential equation of the family of concentric circles $x^2+y^2=a^2$

Find the differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants.

Examples: 9.3 - 4(ii), 6, 10, 11, 15

■ **Solution of Differential equations:**

Solve: $sinx\,\frac{dy}{dx} - y = sinx.\tan\frac{x}{2}$

Solve the following differential equation:

$$x\frac{dy}{dx} + 2y = x^2 logx$$

Examples:

9.4.1 – 3(iii), 5(iii), 7(iii), 8(ii), 9(ii), 12(ii), 15(I, iii), 37, 40

9.4.2 – 2(i), 4(i) **9.4.3 –** 4, 8(ii), 9(i), 13, 14, 15, 20

9.4.4 – 2(i), 4(ii, iii), 6(iii), 11(i), 13(ii), 18(i), 24, 25, 28, 35(i)

Probability:

If A and B are events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$ and P(A∩B) = $\frac{1}{4}$, then find:

(a) $P(A/B)$

(b) $P(B/A)$

In a race, the probabilities of A and B winning the race are $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability of neither of them winning the race.

A speaks truth in 60% of the cases, while *B* in 40% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?

From a lot of 6 items containing 2 defective items, a sample of 4 items are drawn at random. Let the random variable X denote the number of defective items in the sample. If the sample is drawn without replacement, find:

(a) The probability distribution of X

(b) Mean of X

(c) Variance of X

Examples:

10.1 – 5, 6, 9, 11, 12, 13, 15, 21

10.2 – 7, 9, 12, 31

10.3 – 6, 15, 17, 28, 35, 39, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55

10.4 – 1, 5, 9, 13

10.5 – 10, 11, 13, 15, 24, 29, 31

10.7 – 4, 12, 14 **10.8 –** 3, 5, 7, 10, 13, 14, 18

10.9 – 7, 8, 12, 13, 19, 22, 26, 31

10.10 – 2, 6, 9, 11, 15, 16, 17

Section B

Vectors:

Find λ if the scalar projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

If A, B, C are three non-collinear points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, respectively, then show that the length of the perpendicular from C on AB is $\frac{|(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{b}\times\vec{a})|}{|\vec{b}-\vec{a}|}$.

Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

Show that:

$$(\vec{a}\times\vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$$

Show that:

$$\vec{a}.(\vec{b}+\vec{c})\times(\vec{a}+2\vec{b}+3\vec{c}) = [\vec{a} \quad \vec{b} \quad \vec{c}]$$

Other examples:

1.2 – 16, 22, 31, 38, 40 **1.3 –** 21, 23, 27(i), 29, 30, 32, 34, 39, 42, 44, 51, 52, 56, 58

1.4 – 4, 5, 9, 15, 18, 22, 29, 38, 41 **1.5 –** 4(iv,v), 5, 10, 12, 14

Three – Dimensional Geometry:

The Cartesian equation of a line is: $2x - 3 = 3y + 1 = 5 - 6z$. Find the vector equation of a line passing through $(7, -5, 0)$ and parallel to the given line.

Find the equation of the plane through the intersection of the planes

$\vec{r}.(\hat{\imath} + 3\hat{\jmath} - \hat{k}) = 9$ and $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) = 3$ and passing through the origin.

Find the image of a point having position vector : $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ in the Plane $\vec{r}.\left(3\hat{\imath} - \hat{\jmath} + 4\hat{k}\right) = 2$.

Other examples:

2.1 – 24, 26 **2.2 –** 12, 17, 20 **2.3 –** 5, 8, 11 **2.4 –** 1, 6, 8, 9

2.5 – 14, 19, 22, 25, 27, 31 **2.5.4 –** 6, 11, 14, 18, 25, 26, 28, 30

2.6 – 5, 8, 10, 15, 23, 24 **2.7 –** 5, 11, 15, 16

Applications of Integrals:

Draw a rough sketch of the curve and find the area of the region bounded by curve $y^2 = 8x$ and the line $x = 2$.

Sketch the graph of $y = |x + 4|$. Using integration, find the area of the region bounded by the curve $y = |x + 4|$ and $x = -6$ *and* $x = 0$.

Other examples:

3.2 – 2, 4, 7, 8, 9, 14, 16, 18, 19, 25, 29, 30, 32, 33, 37, 47

Section C

Application of Calculus:

Given the total cost function for x units of a commodity as:

$$C(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 2.$$

Find:

(i) Marginal cost function

(ii) Average cost function

The average cost function associated with producing and marketing x units of an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find the range of values of the output x, for which AC is increasing.

A product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$, where x is the number of units produced. The price at which each unit can be sold is given by $P = \left(200 - \frac{x}{400}\right)$. Determine the production level x at which the profit is maximum. What is the price per unit and total profit at the level of production?

A manufacturer's marginal cost function is $\frac{500}{\sqrt{2x+25}}$. Find the cost involved to increase production from 100 units to 300 units.

More examples:

1.3 – 5, 7, 8 **1.4 –** 2, 3, 6, 8, 12, 13, 14

Linear Regression:

Find the coefficient of correlation from the regression lines:

$$x - 2y + 3 = 0 \text{ and } 4x - 5y + 1 = 0.$$

Find the line of regression of y on x from the following table.

x	1	2	3	4	5
y	7	6	5	4	3

Hence, estimate the value of y when $x = 6$.

From the given data:

Variable	x	y
Mean	6	8
Standard Deviation	4	6

and correlation coefficient: $\frac{2}{3}$. Find:

(i) Regression coefficients b_{yx} and b_{xy}

(ii) Regression line x on y

(iii) Most likely value of x when $y = 14$

Other examples:

2.1 - 3, 5, 6, 7, 8, 9, 10, 11, 13, 15

Linear Programming:

A manufacturing company makes two types of teaching aids *A* and ***B*** of Mathematics for Class *X*. Each type of *A* requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of ***B*** requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type *A* and ₹ 120 on each piece of type ***B***. How many pieces of type *A* and type ***B*** should be manufactured per week to get a maximum profit? Formulate this as Linear Programming Problem and solve it. Identify the feasible region from the rough sketch.

A toy company manufactures two types of dolls A and B. Market test and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demands for the dolls of type B is atmost half of that for dolls of type A. Further, the production level of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of ₹ 12 and ₹ 16 per doll respectively on dolls A and B, how many of each type of dolls should be produced weekly, in order to maximise the profit?

Other examples:

3.3 – 15, 17, 25, 26, 27, 33, 35