



ICSE YEAR 2014 MATHEMATICS

- SOLUTION OF 2014
- COMMENTS OF COUNCIL EXAMINERS
- SUGGESTIONS FOR TEACHERS

Dedicated to all my lovely students. May God help you always.

This small booklet contains solution of 2014 ICSE Mathematics. The comments from the council examiners under solution of every question makes this a very handy guide for students to understand what the council expects as answer from the students.

I hope that the students will find this to be useful.

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23rd February, 2015

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MATHEMATICS

Question 1

- (a) Ranbir borrows ₹ 20,000 at 12% per annum compound interest. If he repays ₹ 8400 at the end of the first year and ₹ 9680 at the end of the second year, find the amount of loan outstanding at the beginning of the third year. [3]
- (b) Find the values of x , which satisfy the inequation $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$, $x \in W$. Graph the solution set on the number line. [3]
- (c) A die has 6 faces marked by the given numbers as shown below:
- | | | | | | |
|---|---|---|----|----|----|
| 1 | 2 | 3 | -1 | -2 | -3 |
|---|---|---|----|----|----|
- The die is thrown once. What is the probability of getting
- a positive integer.
 - an integer greater than -3 .
 - the smallest integer.
- [4]

Examiners' Comments

- (a) Most candidates committed a number of calculation errors. Some went on to calculate the amount at the end of the third year instead of the beginning of the third year. Others used formulas to find the amount in 2 years and then subtracted the repayment which is completely an incorrect concept.
- (b) Errors were made in transposing x terms on one side and constants on the other. Many candidates worked out both inequalities simultaneously and hence made major errors both with signs and transpositions, Solutions were not written in the set form, which led to loss of marks. It is necessary to put arrows on both sides of the number line with at least one extra element on each side so as to indicate the continuity of the infinite real number line.

Suggestions for teachers

- Students must be advised to read the question carefully so as to work out the sum in the correct method i.e. find amount at the end of first year (₹22400). Then principal for the second year after repayment (₹22400 - ₹8400=₹14000). Find amount at the end of second year (₹15680) and finally 15680-9680=₹6000 which is the required result.
- It is advisable to solve inequations of the form $-25/6 < 1/2 - 2x/3 \leq 2$, by working with the two Inequalities separately to avoid gross errors. Solutions to be represented by set notation. Property of negative numbers must be clearly explained, e.g. $-3x > -2 \Rightarrow 3x < 2$ and not $3x > 2$.

- (c) Candidates failed to write sample space or listing the probable outcomes. Answers to probability were not given in the simplest form. Some failed to identify the smallest integer from the given set of integers $\{-3, -2, -1, 1, 2, 3\}$.

Suggestions for teachers

– It is necessary to teach students the three basic steps of solving a probability sum.

(i) Listing the total outcomes and favourable outcomes;

(ii) Finding probability by using $P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$;

(iii) Writing the final answer in the simplest form.

MARKING SCHEME

Question 1

(a) $P_1 = ₹20,000$

$$I_1 = \text{Interest for the first year} = \frac{20000 \times 1 \times 12}{100} = ₹2400 \quad [\text{M1}]$$

$$A_1 = \text{Amount at the end of 1}^{\text{st}} \text{ year} = 20000 + 2400 = ₹22400$$

$$\text{Repayment after 1}^{\text{st}} \text{ year} = ₹8400$$

$$P_2 = \text{principal for the 2}^{\text{nd}} \text{ year} = 22400 - 8400 = ₹14000 \quad [\text{A1}]$$

$$I_2 = \text{Interest for the second year} = \frac{14000 \times 1 \times 12}{100} = ₹1680 \quad [\text{M1}]$$

$$A_2 = \text{Amount at the end of 2}^{\text{nd}} \text{ year} = 14000 + 1680 = ₹15680$$

$$\text{Repayment at the end of 2}^{\text{nd}} \text{ year} = ₹9680$$

$$\text{Amount outstanding at the beginning of 3}^{\text{rd}} \text{ year} = 15680 - 9680 = ₹6000 \quad [\text{A1}]$$

(b)

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3}$	$\frac{1}{2} - \frac{2x}{3} \leq 2$
$\frac{2x}{3} < \frac{1}{2} + \frac{17}{6}$	$\frac{1}{2} - 2 \leq \frac{2x}{3}$
$\frac{2x}{3} < \frac{3+17}{6}$	$\frac{1-4}{2} \leq \frac{2x}{3}$
$\frac{2x}{3} < \frac{10}{3}$	$\frac{-3}{2} \leq \frac{2x}{3}$
$x < 5$	$-2\frac{1}{4} \leq x$

correctly transposing all x terms to one side and numbers to the other side [M1] (any one inequation)

$$-2\frac{1}{4} \leq x < 5, x \in W$$

$$\text{Solution} = \{0, 1, 2, 3, 4\}$$

(c) $S = \{1, 2, 3, \dots, 25\}$, $n(s) = 25$

Let A be the event : getting a prime number

$$A = \{2,3,5,7,11,13,17,19,23\}, n(A) = 9, P(\text{Prime No}) = \frac{n(A)}{n(S)} = \frac{9}{25} \quad [\text{B1}]$$

Let B be the event: getting a no. divisible by 3 or 4
 $B = \{3,4,6,8,9,12,15,16,18,20,21,24\}, n(B) = 12, P(B) = \frac{n(B)}{n(S)} = \frac{12}{25} \quad [\text{B1}]$

Let C be the event: getting a factor of 6
 $C = \{1,2,3,6\}, n(C) = 4$
 $P(C) = \frac{n(C)}{n(S)} = \frac{4}{25} \quad [\text{B1}]$

Question 2

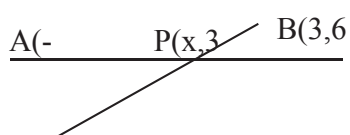
- (a) Find x, y if $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} y \\ 3 \end{bmatrix}$. [3]
- (b) Shahrukh opened a Recurring Deposit Account in a bank and deposited ₹800 per month for $1\frac{1}{2}$ years. If he received ₹15,084 at the time of maturity, find the rate of interest per annum. [3]
- (c) Calculate the ratio in which the line joining A(-4, 2) and B(3, 6) is divided by point P(x, 3). Also find (i) x (ii) Length of AP. [4]

Examiners' Comments

- (a) Errors were made in finding out the product of the 2 by 2 matrix and the 2 by 1 matrix.
 Candidates wrote the product in the form $\begin{bmatrix} 2 & 0 \\ -3 & 2x \end{bmatrix}$ instead of $\begin{bmatrix} 2 + 0 \\ -3 + 2x \end{bmatrix}$
 As a result of incorrect multiplication some candidates failed to evaluate x and y.
- (b) Some candidates considered the given matured value ₹15084 as the interest earned in the given period. A few took the rate as per annum instead of per month. Rate = $r/12$. Some substituted n as $1\frac{1}{2}$ year instead of 18 months.
- (c) Though the question was to find the ratio in which P divides join of A and B, many candidates considered P as the midpoint. Some used section formula correctly but equated to '0' instead of 3 as given in the problem.

Suggestions for teachers

- Concepts of multiplication of 2 matrices should be made clearer, e.g., result of the product of a (2X2) matrix and a (2X1) matrix is a (2X1) matrix.
- It is necessary to stress on reading the question carefully and assimilating the given data before answering it. The given data, e.g., matured value, time, monthly installment etc.
- It is necessary to identify the given data, write them down and then proceed with working with the noted data.

MARKING SCHEME	
Question 2	
Q.2	
(a)	$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} y \\ 3 \end{bmatrix}$ $\begin{bmatrix} 2+0 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$ <p>[M1] [M1]</p> $\begin{bmatrix} -4 \\ 2x \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$ $2y = -4, 2x = 6$ $x = 3, y = -2 \quad \text{[A1] (both correct)}$
(b)	$p = ₹800, n=18, M.V = ₹15084$ $I = \frac{pn(n+1)}{2} \times \frac{1}{12} \times \frac{R}{100} = 800 \times \frac{18 \times 19}{2} \times \frac{1}{12} \times \frac{R}{100} \quad \text{[M1]}$ $= 114R$ <p>Maturity value = $p \times n + \text{Interest}$</p> $15084 = 800 \times 18 + 114R \quad \text{[M1]}$ $R = \frac{15084 - 14400}{114} = \frac{684}{114} = 6\% \quad \text{[A1]}$
(c)	<p>The point of intersection is P(x, 3)</p> $3 = \frac{my_2 + ny_1}{m+n} = \frac{6m+2n}{m+n} \quad \text{[M1]}$ $3m + 3n = 6m + 2n$ $3m = n$ <p>Ratio = $m : n = 1 : 3$ [A1]</p> <p>(i) $x = \frac{1 \times 3 + 3 \times -4}{1+3} = \frac{-9}{4}$ [A1]</p> $\left(\frac{-9}{4}, 3 \right)$ <p>(ii) $AP = \sqrt{\left(-4 + \frac{9}{4}\right)^2 + (2-3)^2} = \sqrt{\frac{49}{16}} + 1 = \sqrt{\frac{65}{16}} = \frac{\sqrt{65}}{4}$ [A1]</p> 

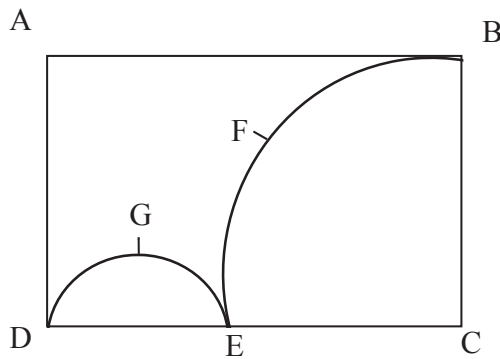
Question 3

- (a) Without using trigonometric tables, evaluate
 $\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$ [3]
- (b) Using the Remainder and Factor Theorem, factorise the following polynomial:

$$x^3 + 10x^2 - 37x + 26.$$

[3]

- (c) In the figure given below, ABCD is a rectangle. AB = 14cm, BC = 7cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle. (Take $\pi = 22/7$)



[4]

Examiners' Comments

- (a) Some candidates adopted incorrect methods of applying complimentary angles
 e.g., $\sin^2 34^\circ = \cos^2(90^\circ - 56^\circ)$
 or, $\sin^2 34^\circ = \sin^2(90^\circ - 34^\circ)$
 The correct method is
 $\sin^2 34^\circ = \cos^2(90^\circ - 34^\circ) = \cos^2 56^\circ$
 Or, $\sin^2 34^\circ = \sin^2(90^\circ - 56^\circ) = \cos^2 56^\circ$
 Many candidates used an incorrect value for $\cot 30^\circ$.
- (b) According to the question it was necessary to use Remainder and Factor Theorem, however some candidates failed to do so and hence lost marks. After factorizing the given mathematical expression, some did not write the result as the product of three factors. i.e.
 $x^3 + 10x^2 - 37x + 26$
 $= (x - 1)(x - 2)(x + 13)$
- (c) π was given to be $22/7$ but some candidates took it as 3.14 hence leading to an incorrect result
 A few candidates took an incorrect radius for the semicircle and the quarter circle. A number of calculation errors were committed by the candidates.

Suggestions for teachers

- The method and concepts of complimentary angles must be made clear with additional revision to avoid the shown errors. It is essential to know the values of the special angles e.g. $\cot 30^\circ$, $\tan 60^\circ$ etc.
- Emphasize on the use of Remainder and Factor Theorem to find at least one factor. It is essential to see that the final result is given as the product of the factors obtained.
- Insist that all given values of a question must be rightly used and deviation from given data would result to inaccurate answers. Students must also be instructed to carefully note the given data, e.g. radius, diameter etc.

$$=14 \times 7 - \left[\frac{1}{4} \cdot \frac{22}{7} \cdot 7.7 + \frac{1}{2} \cdot \frac{22}{7} \cdot \frac{7}{2} \cdot \frac{7}{2} \right]$$

OR

[M1]

$$=98 - [38.5+19.25] \text{ [M1] subtracting sum}$$

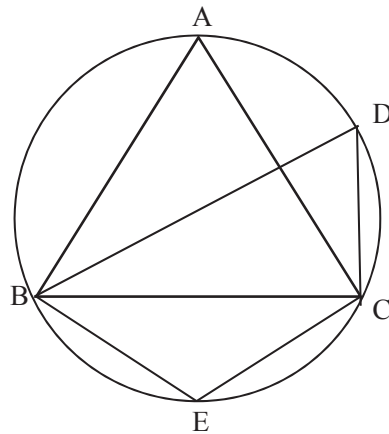
$$= 98 - 57.75 = 40.25 \text{ cm}^2 \text{ [A1]}$$

Question 4

- (a) The numbers 6, 8, 10, 12, 13, and x are arranged in an ascending order.
If the mean of the observations is equal to the median, find the value of x . [3]

- (b) In the figure, $\angle DBC = 58^\circ$. BD is a diameter of the circle. Calculate:

- (i) $\angle BDC$
(ii) $\angle BEC$
(iii) $\angle BAC$



[3]

- (c) Use graph paper to answer the following questions. (Take 2cm = 1 unit on both axis)
- (i) Plot the points $A(-4, 2)$ and $B(2, 4)$
- (ii) A' is the image of A when reflected in the y -axis. Plot it on the graph paper and write the coordinates of A' .
- (iii) B' is the image of B when reflected in the line AA' . Write the coordinates of B' .
- (iv) Write the geometric name of the figure $ABA'B'$.
- (v) Name a line of symmetry of the figure formed. [4]

Examiners' Comments

- (a) In the calculation of mean, conceptual errors were made, e.g. for $49+x$ some candidates took it as $49x$. Method of finding median $\frac{10+12}{2}$ was incorrect. Some did not use the given condition mean=median.
- (b) Some candidates committed errors in this problem for not being clear about the following circle properties:
- Angles in the same segment are equal e.g. $\angle A = \angle D$.
 - Opposite angles of a cyclic quadrilateral are supplementary; e.g. $\angle A + \angle E = 180^\circ$
 - Identification of angle in a semicircle equal to 90° . e.g. $\angle BCD = 90^\circ$ since BD is the diameter.
- In number of cases no proper reasons were stated.
- (c) Most candidates did not use the scale given in the question i.e., 2 cm = 1 unit on both axis. Some marked the coordinate axes incorrectly due to which their plotted points and hence the diagram formed was incorrect. Some made errors in plotting B' (2,0) and hence were unable to name the figure correctly. Some did not join the points $ABA'B'$ to complete the figure.
A majority of candidates did not mark/show the line of symmetry.

Suggestions for teachers

- Basic concepts of algebraic addition needs to be made more clear, e.g., $x+y \neq xy$ or $4+x \neq 4x$. Further the concepts of calculating median of simple distribution needs more drilling.
- The major problem of geometry is that the steps of working are not supported by valid reasons. Teachers must insist on the following:
 - Naming angles correctly e.g., in the figure $\angle ABC$ cannot be written as $\angle B$ as there are two angles at that point.
 - Essential working must be shown;
 - All reasons must be clearly stated.
- Students must be instructed to read their question carefully and follow all given instructions.
Given scales must be used. Stress on correct method of choosing and marking of coordinate axes.
It is essential to write the coordinate of all points plotted and name them according to the given question.
Train students to draw the line of symmetry name and record the particular line for identification e.g. AA'

MARKING SCHEME

Question 4

Q4	
(a)	$\text{median} = \frac{3^{\text{rd}} \text{ term} + 4^{\text{th}} \text{ term}}{2} = \frac{10+12}{2} = 11$ <p style="text-align: center;">Median = Mean</p> $11 = \frac{6+8+10+12+13+x}{6}$ <p style="text-align: center;">$x = 17$</p>

(b)	$\angle DBC = 58^\circ$ $\angle BCD = 90^\circ$ (angle in a semi circle is a rt. $\angle BDC = 180 - (90 + 58^\circ) = 32^\circ$ [M1] $\angle BEC = 180 - 32^\circ$ (opposite angles of a quadrilateral are supplementary) $= 148^\circ$ [A1] $\angle BAC = \angle BDC = 32^\circ$ [A1] (angles in the same segment are equal)		Angle) cyclic
(c)	(i) Plotting points A(-4,2) and B(2,4), on the graph paper [B1] (ii) Reflecting A as A' and writing A' (4,2) OR [B1] (iii) Reflecting B as B' and writing B' (2, 0) (iv) Geometric figure : Kite [B1] (v) AA' is the line of symmetry [B1]		

Question 5

- (a) A shopkeeper bought a washing machine at a discount of 20% from a wholesaler, the printed price of the washing machine being ₹18,000. The shopkeeper sells it to a consumer at a discount of 10% on the printed price. If the rate of sales tax is 8%, find:
- the VAT paid by the shopkeeper.
 - the total amount that the consumer pays for the washing machine. [3]
- (b) If $\frac{x^2 + y^2}{x^2 - y^2} = \frac{17}{8}$, then find the value of:
- $x : y$
 - $\frac{x^3 + y^3}{x^3 - y^3}$ [3]
- (c) In $\triangle ABC$, $\angle ABC = \angle DAC$. $AB = 8\text{cm}$, $AC = 4\text{cm}$, $AD = 5\text{cm}$.
- Prove that $\triangle ACD$ is similar to $\triangle BCA$
 - Find BC and CD
 - Find area of $\triangle ACD$: area of $\triangle ABC$
-

[4]

Examiners' Comments

- (a) Most candidates wrote correct answers by finding the discount and subsequently the consumer's price of the washing machine. A few calculation errors were however committed. Some candidates made calculation errors in finding the VAT paid by the shopkeeper. The concept that VAT is the tax charged on the additional value of an article was not clear.
- (b) Number of errors were detected due to insufficient knowledge of properties of ratio (e.g., compodendo and dividendo).
Some applied comp. & Div. on the one side only, e.g.,

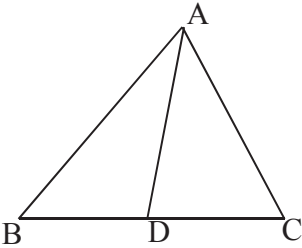
$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}. \text{ But}$$

$$\frac{a}{b} = \frac{c}{d} \not\Rightarrow \frac{a+b}{a-b} = \frac{c}{d}.$$
 In some cases candidates directly wrote $\frac{2x^2}{2y^2} = \frac{25}{9}$, without any steps of working. Some were unable to evaluate $\frac{x^3-y^3}{x^3+y^3}$
- (c) A few candidates took the wrong pair of angles in proving $\triangle ACD \sim \triangle BCA$. In some cases corresponding proportional sides of the two similar triangles was incorrectly written hence found the values of BC and CD.

Suggestions for teachers

- Concepts of VAT must be made clear to the students. VAT = Output Tax – Input tax; Or, Tax charged – Tax paid. Much more emphasis on this concept is essential.
- Students must be thoroughly taught about the properties of ratio and proportion. It is necessary for teachers to emphasize that all essential intermediate steps must be clearly shown.
- Revision is necessary for proving triangles similar and writing down the corresponding proportional sides and area.
- Students must be able to differentiate between congruency and similarity of triangles. It is required to explain that proportional sides are the sides opposite to the corresponding equal angles.

MARKING SCHEME		
Question 5		
(a)	<p><u>Whole saler</u> Printed price = ₹18000, discount = 20% Sales tax = 8% Sales price = price after 20% discount $= 18000 - \frac{20}{100} \times 18000$</p>	<p><u>Shopkeeper</u> \longrightarrow <u>Consumer</u> Printed price = ₹18000, discount = 10% Sales tax = 8% Sales price = price after 10% discount $= 18000 - \frac{10}{100} \times 18000$</p>

	$= 18000 - 3600$ $= ₹14400$ <p>Tax charged by wholesaler = tax paid by Shopkeeper</p> $= \frac{8}{100} \times 14400$ $= ₹1152$	$= 18000 - 1800$ $= ₹16200$ <p>Tax charged by shopkeeper = tax paid by Consumer</p> $= \frac{8}{100} \times 16200$ $= ₹1296$	
	<p>(i) Vat paid by shopkeeper = tax charged by shopkeeper – tax paid by shopkeeper $= 1296 - 1152 = ₹144$ [A1]</p> <p>(ii) Amount at which consumer bought the washing machine = $16200 + 1296$ $= ₹17496$ [A1]</p>		
(b)	<p>(i) $\frac{x^2 + y^2}{x^2 - y^2} = \frac{17}{8}$</p> $\frac{x^2 + y^2 + x^2 - y^2}{x^2 + y^2 - x^2 + y^2} = \frac{17 + 8}{17 - 8}$ [M1] $\frac{2x^2}{2y^2} = \frac{25}{9}$ $\therefore \frac{x}{y} = \frac{5}{3}$ [A1] <p>Or $x : y = 5 : 3$</p>	<p>(ii) $\frac{x}{y} = \frac{5}{3}$</p> $\frac{x^3}{y^3} = \frac{125}{27}$ $\frac{x^3 + y^3}{x^3 - y^3} = \frac{125 + 27}{125 - 27}$ $\frac{x^3 + y^3}{x^3 - y^3} = \frac{152}{98} = \frac{76}{49}$ [A1]	
(c)	<p>(i) In $\triangle ACD$ and $\triangle ABC$, $\angle CAD = \angle ABC$ (given) $\angle ACD = \angle ACB$ ($\angle C$ common) $\triangle ACD \sim \triangle BCA$ (AA test of similarity [B1])</p> <p>(ii) $\frac{AC}{BC} = \frac{AD}{AB} = \frac{CD}{AC}$ or $\frac{4}{BC} = \frac{5}{8} = \frac{CD}{4}$ [M1]</p> <p>$BC = 6.4\text{cm}$ [A1] $CD = 2.5\text{cm}$ $\text{area } \triangle ACD : \text{area } \triangle ABC = 5^2 : 8^2$ $= 25 : 64$ [A1]</p>		

Question 6

- (a) Find the value of 'a' for which the following points A(a, 3), B (2, 1) and C(5, a) are collinear. Hence find the equation of the line. [3]
- (b) Salman invests a sum of money in ₹ 50 shares, paying 15% dividend quoted at 20% premium. If his annual dividend is ₹ 600, calculate:
- (i) the number of shares he bought.

- (ii) his total investment.
 (iii) the rate of return on his investment.
- (c) The surface area of a solid metallic sphere is 2464 cm². It is melted and recast into solid right circular cones of radius 3.5cm and height 7cm. Calculate:
- (i) the radius of the sphere.
 (ii) the number of cones recast. (Take $\pi = 22/7$) [4]

Suggestions for teachers

- It is necessary to explain to the students that they cannot assume a condition for a question if it is not mentioned. Students must focus on the conditions which are given in the question before working it out.
- More practice on concepts related to premium and discount is necessary.
- Students must be advised that no answer must be kept in improper fraction. e.g. $\frac{25}{2}$ must be written as $12\frac{1}{2}$ or 12.5.
- It is necessary to reinforce students with formulas related to volume and surface area. Students must be instructed to use the value of $\pi = 22/7$, if it is given in the question and not $\pi = 3.14$. Instead of finding volume of cone and sphere separately students may be advised to use, **Volume of Sphere = n × volume of cones where n = number of cones.**

Examiners' Comments

- (a) Concepts of co linearity of points was incorrectly used. Many candidates considered B(2,1) as the midpoint of A(a,3) and C(5,a) hence lost marks. Some lacked the application of finding the equation of a line when two or more points are given. They may use $y = mx + c$ or $(y - y_1) = m(x - x_1)$ or any other equivalent form of equation. Some used distance formula which is an extremely long method for this particular sum and hence were unable to calculate the correct answer. To solve the question candidates had to find the slopes between the given points and equate them.
- (b) Some candidates made calculation errors hence wrote incorrect answers. Some were unable to find the market value, resulting in incorrect calculations leading to incorrect answers.
- (c) Some candidates used the wrong formula for surface area of sphere, hence radius of sphere found was incorrect. This led to incorrect number of cones. Some took $\pi = 3.14$ instead of $22/7$ as given in the problem. To calculate correctly the given data must be used.

MARKING SCHEME
Question 6

(a)	<p>A(a, 3), B(2, 1) and C(5, a)</p> <p>Slope of AB = $\frac{1-3}{2-a} = \frac{-2}{2-a}$ OR</p> <p>Slope of BC = $\frac{a-1}{5-2} = \frac{a-1}{3}$ [M1]</p>
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	$\therefore \frac{-2}{2-a} = \frac{a-1}{3}$ $-6 = 3a - a^2 - 2$ $a^2 - 3a - 4 = 0$ $(a-4)(a+1) = 0$ $\therefore a = 4, a = -1. \text{ [A1]}$ <p>When A(4, 3), B(2, 1) ; When A(-1, 3), B(2, 1)</p> $\text{eqn } y - 3 = \frac{1-3}{2-4}(x-4) \qquad \qquad \qquad \text{eqn } y - 3 = \frac{1-3}{2+1}(x+1)$ $\text{or } x - y = 1. \longrightarrow \text{ [A1] } \longleftarrow \qquad \qquad \qquad \therefore 2x + 3y = 7$ <p style="text-align: center;">OR</p>
(b)	<p>Face value = ₹ 50, div% = 15%, M.V. = $50 + \frac{20}{100} \times 50 = ₹ 60$</p> $\text{Annual Dividend} = \frac{n \times 50 \times 15}{100}$ $600 = \frac{n \times 15}{2}$ <p>(i) No. of shares = $n = 80$ [B1] Investment = $n \times \text{M.V. of 1 share} = 80 \times 60$ = ₹ 4800 [B1]</p> <p>(ii) Rate of return = $\frac{\text{A.D.}}{\text{investment}} \times 100 = \frac{600}{4800} \times 100$ = 12.5%</p> <p>Or = $\frac{\text{FV}}{\text{MV}} \times \text{div\%} = \frac{50}{60} \times 15\% = 12.5\%$ [B1]</p>
(c)	<p>surface area of sphere = $4\pi R^2$</p> $2464 = 4 \times \frac{22}{7} \times R^2 \quad \text{[M1]}$ $R^2 = 196$ $R = 14 \text{ cm} \quad \text{[A1]}$ <p>No. of cones = $\frac{\text{Volume of sphere}}{\text{Volume of cone}}$</p> $= \frac{\frac{4}{3} \pi R^3}{\frac{1}{3} \pi r^2 h} = \frac{\frac{4}{3} \times \pi \times 14^3}{\frac{1}{3} \times \pi \times (3.5)^2 \times 7} \quad \text{[M1]}$ $= 128 \text{ cones} \quad \text{[A1]}$

Question 7

(a) Calculate the mean of the distribution given below using the short cut method.

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of students	2	6	10	12	9	7	4

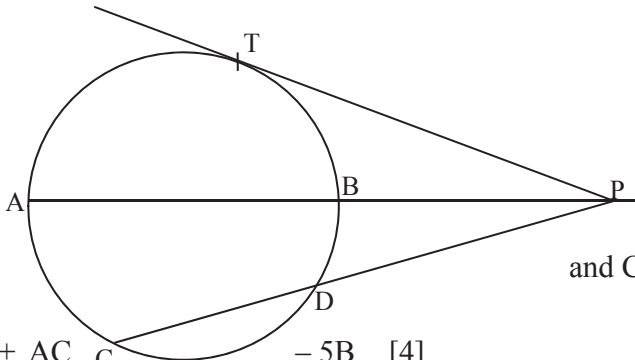
[3]

(b) In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8cm, PD = 5cm, PB = 4cm. Find:

(i) AB.

(ii) the length of tangent PT.

[3]

(c) Let $A =$  $\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$
and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$.

Find $A^2 + AC - 5B$. [4]

Examiners' Comments

(a) Most candidates did not use the shortcut method to calculate mean.

It is essential to find the class mark of a group frequency distribution irrespective of the method adopted to calculate its mean. e.g. for direct method we find fx , short cut method $d=x-A$ and step deviation $t = \frac{x-A}{i}$ and in each case 'x' represents the class mark.

(b) Instead of using $PT^2 = PA \times PB$, or $PT^2 = PC \times PD$ some candidates wrote $PT^2 = PB \times AB$ or $PT^2 = PD \times CD$. Hence they committed errors resulting in wrong answers. Some correctly proved triangles similar and found the correct answers.

(c) Many candidates made errors in finding the product A^2 or AC . Only a few found A^2 by squaring each element of the matrix A. Some made calculation errors especially those that involved the negative sign.

Suggestions for teachers

- Emphasize on all three methods of calculation of mean.
- For convenience of working guide students to choose one of the class mark as the assumed mean.
- The various properties of circles in Geometry needs repeated drilling so as to find unknown values using given conditions. Reasons must be clearly stated.
- More revision of multiplication of two matrices is essential. Stress must also be laid on Matrix addition and subtraction.

MARKING SCHEME

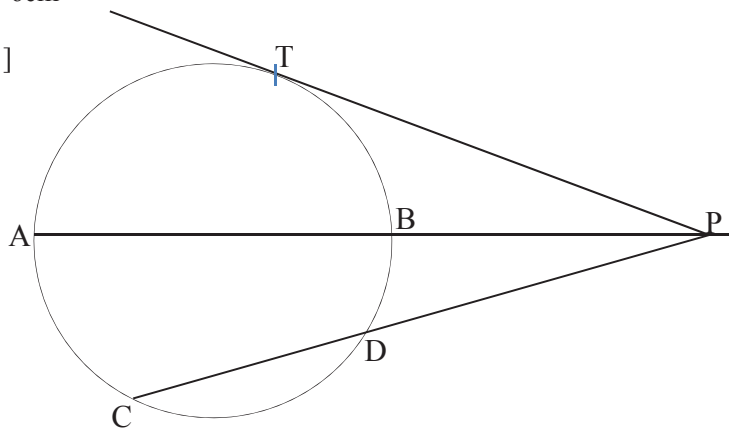
Question 7

Marks	No. of students	x	d= x- A	fd
11 – 20	2	15.5	- 30	- 60
21- 30	6	25.5	- 20	-120
31- 40	10	35.5	- 10	- 100
41- 50	12	45.5	0	0
51- 60	9	55.5	10	90
61- 70	7	65.5	20	140
71- 80	4	75.5	30	120
Total	$\sum f = 50$			$\sum fd = 70$

[M1] for x or d column any 4 correct

Assumed mean = A = 45.5, Mean = $A + \frac{\sum fd}{\sum f} = 45.5 + \frac{70}{50}$ [M1]
 $= 45.5 + 1.4 = 46.9$ [A1]

(b) PA x PB = PC x PD
 PA x 4 = 12.8 x 5 [M1]
 $PA = \frac{12.8 \times 5}{4} = 16cm$
 AB = 16 – 4 = 12cm [A1]
 Radius of the circle = 6cm
 $PT^2 = PA \times PB = 16 \times 4$
 $PT = \sqrt{64} = 8cm$ [A1]



(c) $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}, C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$
 $A^2 + AC - 5B$
 $= \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} - 5 \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$
 $= \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} + \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$
 [M1] [M1] [M1]
 $= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 8 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix} \quad [A1]$$

Question 8

- (a) The compound interest, calculated yearly, on a certain sum of money for the second year is ₹1320 and for the third year is ₹1452. Calculate the rate of interest and the original sum of money. [3]
- (b) Construct a ΔABC with $BC= 6.5$ cm, $AB= 5.5$ cm, $AC=5$ cm. Construct the incircle of the triangle. Measure and record the radius of the incircle. [3]
- (c) (Use a graph paper for this question.) The daily pocket expenses of 200 students in a school are given below:

Pocket expenses (in ₹)	Number of students (frequency)
0 - 5	10
0 - 10	14
10 - 15	28
15 - 20	42
20 - 25	50
25 - 30	30
30 - 35	14
35 - 40	12

Draw a histogram representing the above distribution and estimate the mode from the graph. [4]

Examiners' Comments

- (a) Many candidates were unable to differentiate between the concept of CI in 2 or 3 years and CI for the 2nd or 3rd year. Here interest for the 2nd and third year were ₹1320 and ₹1452 respectively. However answers was incorrect calculated. Some found the rate correctly but were unable to find the original value.
- (b) Errors were made by candidates for using incorrect measurements of AB, BC, AC to construct triangle ABC. Several candidates did not drop a perpendicular on one of the sides from the point of intersection of the bisectors of the angles so as to get the radius of the in circle.

Suggestions for teachers

- The difference of CI in 2 years and CI during the 2nd year must be made clear to the students. Sufficient practice of inverse problems of CI is essential for better performance.
- Students must be advised to show all traces of construction in Geometry. And all given measurements must be correctly taken.
- Emphasize more on scale and choice of axes for all graphs.
- Adequate revision is necessary to read correct values from the graph.

- (c) Common errors committed in this particular question are as follows:
- Incorrect axis. The gaps between the class intervals were not uniformly marked.
 - Failure to draw the necessary intersecting lines and dropping a perpendicular to identify mode.
 - Errors were made by candidates in recording the value from the graph.

MARKING SCHEME

Question 8

(a)

$$\text{Rate of Interest} = \frac{I_3 - I_2}{I_2} \times 100 = \frac{1452 - 1320}{1320} \times 100 \quad [\text{M1}]$$

$$= \frac{132}{1320} \times 100 = 10\% \quad [\text{A1}]$$

Let Principal be ₹ 100

$$I_1 = \frac{100 \times 1 \times 10}{100} = 10$$

$$I_2 = \frac{110 \times 1 \times 10}{100} = 11$$

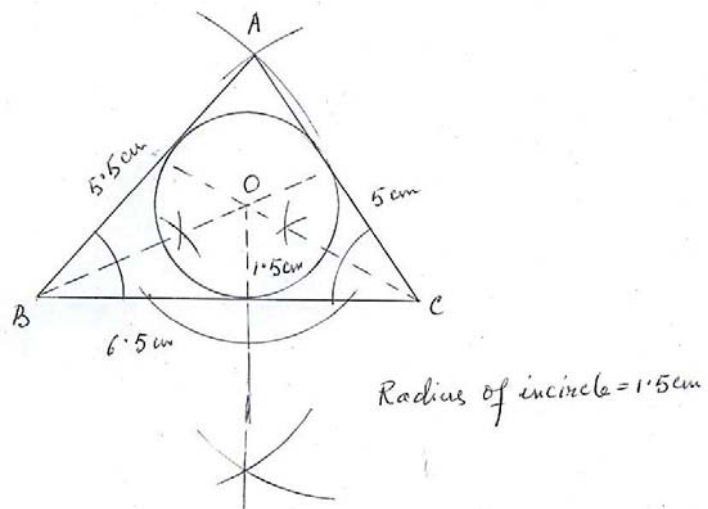
If the P = 100 , $I_2 = 11$
 $x = ?$, $I_2 = 1320$

$$\text{Sum of money} = x = \frac{1320 \times 100}{11}$$

$$= ₹12000 \quad [\text{A1}]$$

(b)

Constructing ΔABC [M1]
 Constructing the angle bisectors of 2 angles and getting the incircle. [M1]
 Radius of the incircle = 1.5 cm ($\pm .2$) [A1]



(c)	Pocket expenses (in ₹)	Number of students (frequency)	
	0 - 5	10	Correct Bars [M1]
	0 - 10	14	Correct x and y-axis [B1]
	10 - 15	28	Lines for mode [M1]
	15 - 20	42	Mode = 21.4 (±.6) [A1]
	20 - 25	50	
	25 - 30	30	
	30 - 35	14	
	35 - 40	12	

Question 9

- (a) If $(x - 9) : (3x + 6)$ is the duplicate ratio of $4 : 9$, find the value of x . [3]
- (b) Solve for x using the quadratic formula. Write your answer correct to two significant figures. $(x - 1)^2 - 3x + 4 = 0$. [3]
- (c) A page from the savings bank account of Priyanka is given below:

Date	Particulars	Amount withdrawn (₹)	Amount deposited (₹)	Balance (₹)
03/04/2006	B/F			4000.00
05/04/2006	By cash		2000.00	6000.00
18/04/2006	By cheque		6000.00	12000.00
25/05/2006	To cheque	5000.00		7000.00
30/05/2006	By cash		3000.00	10000.00
20/07/2006	By self	4000.00		6000.00
10/09/2006	By cash		2000.00	8000.00
19/09/2006	To cheque	1000.00		7000.00

If the interest earned by Priyanka for the period ending September, 2006 is ₹ 175, find the rate of interest. [4]

Examiners' Comments

- (a) Some candidates failed to understand the concept of duplicate ratio hence were unable to solve the problem. Some made errors in considering 4:9 as the duplicate ratio of $(x-9) : (3x+6)$.
- (b) Some candidates wrote $(x-1)^2$ as x^2-1 instead of x^2-2x+1 hence lost marks.

Candidates while substituting for $\sqrt{5}$ rounded off at the initial stage hence did not arrive at the correct answer. All approximations as stated in the question must be carried out at the final step. e.g. in this sum after division answer is 3.618 and 1.382. Hence the answer correct to two significant figures is 3.6 and 1.4.

- (c) Some candidates made mistakes in identifying the minimum balances for some of the months. This led to incorrect answer. Some took rate as R and not R/12. Since the problem involves interest for 6 months a few took $t = 6$ instead of 1.

Suggestions for teachers

- Basic concepts of ratio must be made clear to the students, e.g. duplicate, triplicate, sub duplicate ratio etc.
- Recapitulation old concepts like $(a \pm b)^2$ etc. is necessary. Further, concepts of approximation of all types need thorough drilling.
- Teach students to find square root using log tables. This helps them to save time in finding the square root.
- Concepts of finding interest using $\frac{P \times R \times 1}{100 \times 12}$ must be made clear, i.e. $t = 1$ as interest is calculated monthly and divided by 12 as rate given is per annum.

MARKING SCHEME	
Question 9	
Q.9 (a)	$\frac{x-9}{3x+6} = \left(\frac{4}{9}\right)^2$ <p style="text-align: right;">[M1]</p> $81(x-9) = 16(3x+6)$ $81x - 729 = 48x + 96$ <p style="text-align: right;">[M1]</p> $33x = 825$ $X = 25$ <p style="text-align: right;">[A1]</p>
(b)	$(x-1)^2 - 3x + 4 = 0$ $x^2 - 2x + 1 - 3x + 4 = 0$ $x^2 - 5x + 5 = 0$ <p style="text-align: right;">[M1]</p> $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 5}}{2}$ <p style="text-align: right;">[M1]</p> $= \frac{5 \pm \sqrt{25 - 20}}{2}$ $= \frac{5 \pm \sqrt{5}}{2} = \frac{5 \pm 2.236}{2}$ $x = \frac{5 + 2.236}{2}; x = \frac{5 - 2.236}{2}$

$$= \frac{7.236}{2}; \quad x = \frac{2.764}{2}$$

$$= 3.618 \quad = 1.382$$

$$\therefore x = 3.6, \quad = 1.4 \text{ [A1] (Both correct)}$$

(c)

Month	Minimum balance (₹)
April	6,000
May	7,000
June	10,000
July	6,000
August	6,000
September	7,000
Total	P= 42,000

[M1] for any 4 minimum balance correct

$$P = 42,000, \quad \text{[M1] (all correct)}$$

$$I = 175,$$

$$S.I. = P \times \frac{1}{12} \times \frac{R}{100}$$

$$175 = \frac{42000 \times 1 \times R}{12 \times 100} \quad \text{[M1]}$$

$$R = \frac{175 \times 1200}{42000} = 5\% \quad \text{[A1]}$$

Question 10

(a) A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. [4]

(b) The marks obtained by 100 students in a Mathematics test are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No of students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.

Use a scale of 2cm = 10 units on both axis).

Use the ogive to estimate the:

(i) median.

(ii) lower quartile.

(iii) number of students who obtained more than 85% marks in the test.

(iv) number of students who did not pass in the test if the pass percentage was 35. [6]

Examiners' Comments

- (a) A few candidates failed to identify the two digit number as $10x+y$ or $10 \times \frac{6}{x} + x$ and hence were unable to form the right quadratic equation. Some calculations were based on a trial and error method without offering any proper explanations.
- (b) Errors were made in finding CF. Candidates failed to tally the last CF with the total frequency 100. Scale was not taken according to data given in the question. Lines were not shown for identifying the median, quartiles etc. Some candidates selected the x and y axis in the reverse order due to which the orientation of the S curve changed.

Suggestions for teachers

- For a quadratic equation problem it is essential to form a quadratic equation with the given conditions and hence solve to find the unknown number.
- It must be noted that ogive is a cumulative frequency curve and the plotted points must be joined freehand and not with ruler. Students must be advised to use a scale as advised in the question. Revision and supervision is necessary to avoid such errors.

MARKING SCHEME				
Question 10				
(a)	<p>Let the unit's digit be x, ten's digit = $\frac{6}{x}$</p> <p style="text-align: center;">Original no. = $10 \times \frac{6}{x} + x = \frac{60}{x} + x$ ← [B1] (any one correct)</p> <p style="text-align: center;">Reversed no. = $10x + \frac{6}{x}$ ←</p> <p>Original no + 9 = Reversed no.</p> $\frac{60}{x} + x + 9 = 10x + \frac{6}{x} \quad \text{[M1] equation}$ $9 = 9x + \frac{-54}{x}$ $1 = x - \frac{6}{x}$ $x^2 - x - 6 = 0 \quad \text{[M1]}$ $(x - 3)(x + 2) = 0$ <p>X = -2 (not possible) , x = 3</p> <p>Original number = $\frac{60}{x} + x = \frac{60}{3} + 3 = 23$ [A1]</p>			
(b)		Marks	Frequency(f)	c.f.
		0 -10	3	3
		10-20	7	10
		20-30	12	22

30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

[B1] (for first 6 c.f. correct)

[B1] (for smooth curve plotted with upper boundaries)

$N = 100$

(i) Median = $\frac{N}{2}$ th term, = 50th term. = 45 (± 1) [A1]

(ii) lower quartile = $\frac{N}{4}$ th term = 25th term 32 (± 1) [A1]

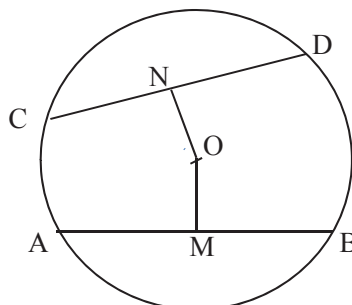
(iii) No. of students who obtained more than 85% marks. = 6 (± 1) [A1]

(iv) No. of students who did not pass the test. = 29 (± 1) [A1]

Question 11

- (a) In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD. AB = 24cm, OM = 5cm, ON = 12cm. Find the:

- (i) radius of the circle.
(ii) length of chord CD.



[3]

- (b) Prove the identity

$$(\sin \theta + \cos \theta) (\tan \theta + \cot \theta) = \sec \theta + \operatorname{cosec} \theta.$$

[3]

- (c) An aeroplane at an altitude of 250 m observes the angle of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river.

Write the answer correct to the nearest whole number.

[4]

Examiners' Comments

- (a) Most candidates answered the question correctly.
Some found CN but did not find the value of CD.
- (b) Some common errors observed were :
(i) Working with both sides together; (ii) Skipping of necessary steps so as to get the answer; (iii) some opened the LHS expression but failed to simplify and come to the RHS.
- (c) Some candidates did not draw the diagram hence lost marks. Some used $\tan 60^\circ = 1.732$ instead of $\sqrt{3}$ for which working consumed extra time. A few did not calculate the answer to the nearest whole number.

Suggestions for teachers

- Reasons to Geometry problems are essential and this needs regular drilling and supervision.
- Ensure that while proving identities students proceed with either LHS or RHS but must not work with both sides simultaneously.
- Advise students to draw a labeled diagram for all height and distance problem following the given conditions. For values like $\tan 60^\circ$ students must consider it as $\sqrt{3}$ and not 1.732 else working is long and tedious. Students must be advised to read question carefully so as to avoid missing out the sub parts of the question specially rounding off of answers.

MARKING SCHEME

Question 11

Q.11 AM = BM = $\frac{1}{2}$ (24) (perpendicular from centre of a circle bisect the chord.)

(a)

$$= 12 \text{ cm}$$

From ΔAOM , $AO^2 = 12^2 + 5^2 = 169$ [M1]

Radius of the circle = $\sqrt{169} = 13 \text{ cm}$ [A1]

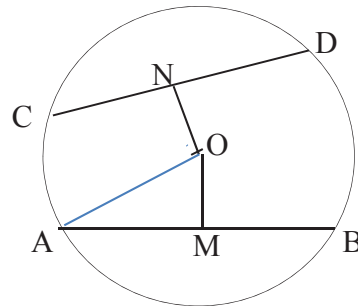
CO = AO = 13 cm

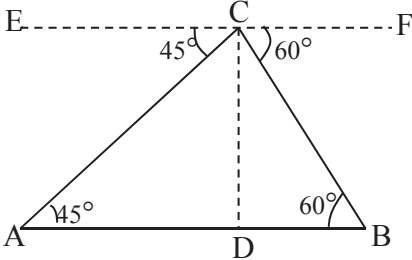
From ΔCON , $13^2 = 12^2 + CN^2$

$$CN^2 = 169 - 144 = 25$$

CN = 5cm,

Chord CD = $2 \times 5 = 10 \text{ cm}$ [A1]



(b)	$\begin{aligned} \text{LHS} &= (\sin \theta + \cos \theta)(\tan \theta + \cot \theta) \\ &= (\sin \theta + \cos \theta) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) && \text{[M1]} \\ &= (\sin \theta + \cos \theta) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) \\ &= (\sin \theta + \cos \theta) \left(\frac{1}{\cos \theta \sin \theta} \right) && \text{[M1]} \\ &= \frac{1}{\cos \theta} + \frac{1}{\sin \theta} \\ &= \sec \theta + \operatorname{cosec} \theta = \text{RHS} && \text{[A1]} \\ &\text{LHS} = \text{RHS} \end{aligned}$
(c)	<p>CD = AD = 250 m [M1] ($\angle A = 45^\circ$)</p> <p>$\tan 60^\circ = \frac{CD}{DB}$</p> <p>[B1] $\sqrt{3} = \frac{250}{DB}$ [M1]</p> <p>$\therefore DB = \frac{250}{\sqrt{3}} = \frac{250 \times \sqrt{3}}{3}$</p> <p>$= \frac{250 \times 1.732}{3} = 144.33$</p> <p>$\therefore AB = 250 + 144.33 = 394.33$</p> <p>Ans = 394 m [A1] CAO</p> 

Topics/Concepts Found Difficult

- Value Added Tax (VAT)
- Compound Interest inverse problems.
- Trigonometry
- Similarity
- Rounding off final result e.g. significant figures.
- Theorems on properties of circle.
- Properties of proportion.
- Construction of incircle.
- Short cut method of calculation of mean
- Coordinate geometry, Section formula and identifying points on x or y axis. Conditions of collinear.
- Quadratic equation problem

Suggestions for Candidates

- Reading time must be utilized to make the right choice of questions and make oneself familiar with all given data
- More practice must be done on rounding off of digits
- Use graph paper for questions based on graphs
- Use of log table to find square root of numbers
- Avoid skipping steps. All necessary steps must be clearly shown
- Working for matrix multiplication is essential
- Adopt methods where lesser calculation is necessary to get final result
- Necessary sample space must be written for probability problems.
- Steps of working is necessary in conversion of trigonometric ratio's of complementary angles.
- Reasons must be provided for all geometry problems.